

I-SEEC2011

Analysis of Thermoelectric Generator by Finite Element Method

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Elsevier use only: Received 30 September 2011; Revised 10 November 2011; Accepted 25 November 2011.

Abstract

This paper focuses on designing thermoelectric generator (TEG) via numerical method. Thermoelectric module in this study was made from n-type CaMnO_3 and p-type $\text{Ca}_3\text{Co}_4\text{O}_9$. The couple field equations were formulated into a system of algebraic equations by means of finite element method (FEM). The simulation procedure was verified by comparing with experimental results, which showed a very good agreement between the two outcomes. The parametric design exhibited that output voltages vary linearly with height of TEG, until reaching some limiting value, the linear relationship was terminated. After that specific height, increasing the height of TEG no longer gains notable output voltage.

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Keywords: thermoelectric generator; finite element method; numerical simulation; geometric design

1. Introduction

The development of materials science with good thermoelectric properties made achievable the fabrication of thermoelectric generators (TEG) [1-3]. Being sophisticated devices, thermoelectric systems

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offer some unique advantages, such as high reliability, long life, small-size, environmentally-green and no-vibrations generator, and can be used in a wide temperature range. Thermoelectric generators are based on the Seebeck effect for power generation. If a steady temperature gradient is applied along a conducting sample, the initially uniform charge carrier distribution is disturbed as the free carriers located at the high-temperature end diffuse to the low-temperature end. This results in the generation of a back electromotive force which opposed any further diffusion current. The open-circuit voltage when no current flows are the Seebeck voltage. When the junctions of a circuit formed from two dissimilar conductors (n- and p-type semiconductors) connected electrically in series but thermally in parallel are maintained at different temperatures $T_{\text{hot}} = T_1$ and $T_{\text{cool}} = T_2$ the open-circuit voltage V is developed. The thermoelectric generator is a standardized device consisting of p- and n-type legs connected electrically in series and thermally in parallel, and bonded to a ceramic plate on each side. The modules are fabricated in a great variety of sizes, shapes and number of thermoelectric couples and can operate in a wide range of temperature gradient. When a temperature gradient is applied across the thermoelectric generator, the heat absorbed at the hot junction and cold side will generate a current through the circuit and deliver electrical power to the load resistance. This study aims to find out the effect of geometric design to the thermoelectric generator. The typical models of square-cross-section area TEG associated with various heights are under investigation. By assigning area to constant, but increase height value, we could come upon the optimal height which issue highest power output. This process is very helpful to design real thermoelectric devices for commercial products.

2. Computational Details

The governing equations to describe behaviour of thermoelectric material are governed by the couple equations of heat transfer and continuity of current density phenomena as follow [4]:

$$\rho c \frac{\partial T}{\partial t} + \vec{\nabla} \cdot \vec{q} = Q \quad (1)$$

$$\vec{\nabla} \cdot \left(\varepsilon \frac{\partial \vec{E}}{\partial t} \right) + \vec{\nabla} \cdot \vec{J} = Q \quad (2)$$

Where T is temperature, \vec{E} is electric field, \vec{q} is heat flux, \vec{J} is electrical current density, Q is internal heat generator, t is time, ρ is density, c is heat capacity and ε is electric permittivity. The current density \vec{J} is generated by a coupling of reversible Seebeck effect and irreversible Joule effect can be written as

$$\vec{J} = \sigma \vec{E} - \sigma S \vec{\nabla} T \quad (3)$$

The heat flux \vec{q} is generated by couple effect of reversible Peltier and irreversible Fourier effect is defined as

$$\vec{q} = \Pi \vec{J} - \kappa \vec{\nabla} T \quad (4)$$

where Π is Peltier coefficient, κ is thermal conductivity, σ is electrical conductivity and S is Seebeck coefficient.

The electric field \vec{E} can be derived from an electric scalar potential ϕ as

$$\vec{E} = -\vec{\nabla} \phi \quad (5)$$

The two coupled governing equations (1) and (2) are then transformed into finite element equations by approximating the primitive physical unknowns, temperature T and electrical potential ϕ , into interpolation functions and value of nodal known on a element by

$$T = [N] \{T_e\} \quad (6)$$

$$\phi = [N] \{\phi_e\} \quad (7)$$

where T_e is vector of nodal temperature, ϕ_e is vector of nodal electrical potential and N is element shape functions. After lengthy manipulations base on the Galerkin weighting scheme, the differential equations become algebraic finite element equations [5-7],

$$\begin{bmatrix} C_T & 0 \\ 0 & C_E \end{bmatrix} \begin{bmatrix} \frac{\partial T^2}{\partial t} \\ \frac{\partial \phi^2}{\partial t} \end{bmatrix} + \begin{bmatrix} K_T & 0 \\ K_{ET} & K_E \end{bmatrix} \begin{bmatrix} T^e \\ \phi^e \end{bmatrix} = \begin{bmatrix} Q \\ I \end{bmatrix} \quad (8)$$

The global matrix equation is assembled from the individual finite element equations. The solution yields temperatures, T_e , and electric potentials, ϕ_e , at unconstrained nodes, or reactions in the form of heat flow rate and electric current at nodes with imposed temperature and electric potential respectively

Thermoelectric module is made of CaMnO_3 and $\text{Ca}_3\text{Co}_4\text{O}_9$ for n-and p-type leg, respectively. The module is fabricated by connect n-and p-cells with thin copper plates and superimposed alumina plates on top and bottom sides of its structure, as shown in Fig. 1(a) and 1(b). To establish the electrical contact between the cells, the copper and alumina plate are served as substrate and supporting floor. The n-and p-leg was similar dimension of $3 \times 3 \times 4 \text{ mm}^3$. Notice that the height of n-and p-leg are 4 mm. The copper plate and alumina plate have size of $5 \times 8 \times 0.3 \text{ mm}^3$ and $5 \times 10 \times 0.7 \text{ mm}^3$, respectively.

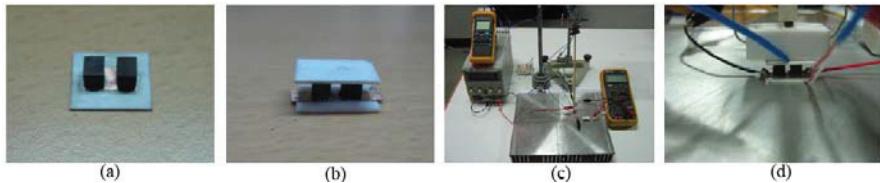


Fig. 1. (a) & (b) Fabrication of a single module TEG, (c) & (d) The laboratory equipment set-up for measure output voltage obtained from differential temperature on the TEG

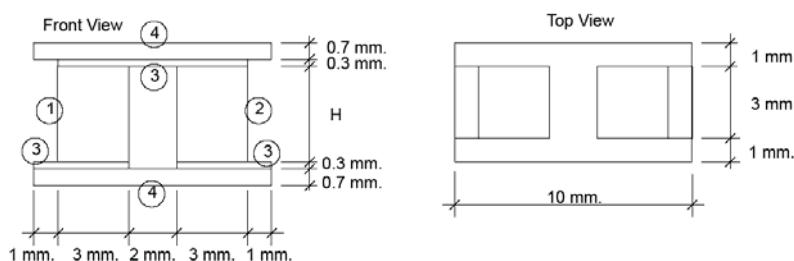


Fig. 2. Geometric detail of TEG

Table 1. Material properties

Materials	S [$\mu\text{V/K}$]	κ [$\text{W}\cdot\text{m}^{-1}$]	α [$\text{W/m}\cdot\text{K}$]
$\text{Ca}_3\text{Co}_4\text{O}_9$	147.31	9.78×10^{-5}	8.55
CaMnO_3	-480.23	12.50×10^{-5}	9.79
Copper	6.50	1.69×10^{-8}	350
Alumina	–	1.00×10^{14}	40

Thermoelectric structure was activated by differential temperature varying in range 0-40 K. The output voltage generated was measured by the instruments as presented in Fig. 1c and 1d. As visualized by the plotting results in Fig. 4(a), output voltage related to driven temperature as linear function exhibit by a straight line.

3. Results and Discussion

Numerical simulation is carried out by using a finite element package ANSYS. The procedure for doing a thermoelectricity analysis consists of following main steps: create the physics environment, build and mesh the model and assign physics attributes to each region within the model, apply boundary conditions and loads excitation, obtain the solution, review the results.

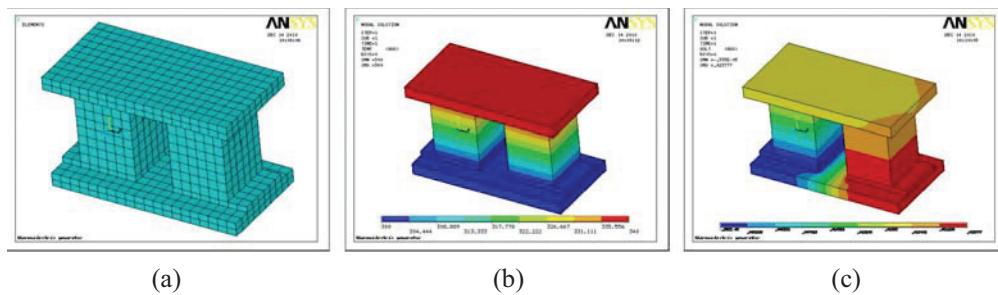


Fig. 3. (a) Computational mesh, (b) Temperature distribution, (c) Voltage distribution

Simple thermoelectric generator geometry consists of n-type CaMnO_3 and p-type $\text{Ca}_3\text{Co}_4\text{O}_9$. Geometric detail of TEG is shown in Fig. 2. Number encompassed by a little circle prescribed material type: 1 = n-type leg, 2 = p-type leg, 3 = copper, 4 = alumina. The material properties for the calculations with temperature independent values are shown in Table 1. In the present application for modelling the electric and thermal fields the second order brick element was chosen. There are 20 nodes per element. Each node consists of 2 unknowns, temperature and voltage.

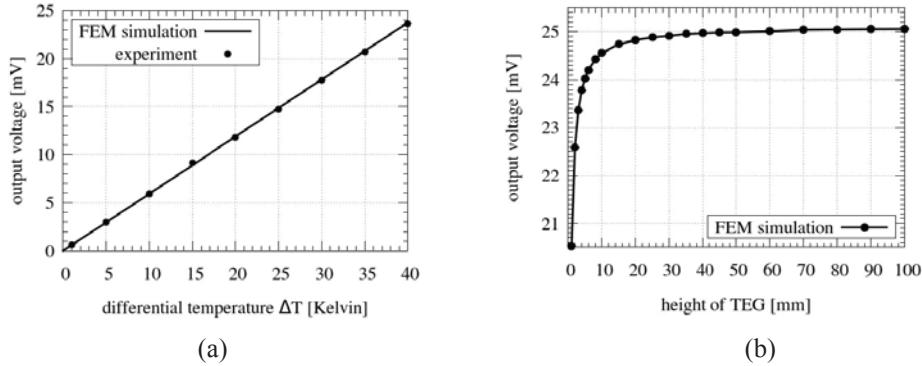


Fig. 4. A comparison of output voltage between the experiment and FEM simulation and output voltage vs. height of TEG

First of all, we verify the computational procedure by comparing numerical results with the experiment in section III. Fig. 4(a) shows good agreement between results from laboratory test and FEM solution. Variation of the voltage output varies against differential temperature related in linear relationship. The more differential temperature results in the more voltage output. The maximum voltage 23.76 mV was obtained when applied 40 K differential temperatures. Distribution of temperature and voltage also are show in Fig. 3(b) and 3(c), respectively.

To study effect of geometric design, we characterize thermoelectric generator into various height. The height of the module varies between 0-10 cm. The results of output vary against height were plotted in Fig. 4(b). From the figure we obviously found that there are 3 regions of output voltage. Firstly, when height varies increasingly from 0 to 6 mm, output voltage has linear relationship with height. Secondly, when height varies increasingly from 6 mm to 40 mm, output voltage still increasing but the slope of the curve was drop. Finally, when the height varies is increasing from 40 mm to 100 mm, output voltage do not increasing but exhibit saturation state. The numerical results suggested that we can use Fig. 4(b) in a design process of TEG. This guideline leads to optimum design. The amount of material use in fabrication of TEG could be economical by using prior results from computational simulation.

4. Conclusions

In this paper, we use numerical technique to study behaviour of thermoelectric generator. The governing equations were transformed into algebraic equations via FEM discretization technique. In this study, we analyzed a single module of thermoelectric generator which made from n-type CaMnO_3 and p-type $\text{Ca}_3\text{Co}_4\text{O}_9$. Laboratory experiment was performed by using TEG with 4 mm. Height. The results from experimental test shown that output voltage vary linearly with differential temperature. Simulation results from FEM were closely agreed with the test. The maximum voltage 23.76 mV was obtained when applied 40 K differential temperatures. We studied effect of geometric design by using a TEG with constant cross section area of $3 \times 3 \text{ mm}^2$, but variable heights. We found that, when increase height, output voltage also increase in linearly fashion until reach a specific height, output voltage was saturated. After the saturated point, stretched more height do not gain more explicit output power. This information suggested that there is some height which produced optimal output voltage.

Acknowledgements

Financial support was provided by the Electricity Generating Authority of Thailand, EGAT (52-2115-043-JOB No. 803-SNRU).

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